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ORBIT DETERMINATION AND ANALYSIS OF COSMOS 220 ROCKET AT RESONANCE, TO EVALUATE 29TH-ORDER HARMONICS

by

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SUMMARY

The orbital parameters of Cosmos 220 rocket (1968-40B) have been determined at 74 epochs from some 6300 observations. For 38 of the determinations, Hewitt camera observations were available.

The orbit was determined from January 1983 until July 1985, and during this time the satellite passed through 29:2 resonance. The variations in inclination and eccentricity have been analysed to determine six lumped geopotential harmonics of order 29. The first two have standard deviations equivalent to accuracies of 0.3 and 0.6 cm in geoid height, better than has previously been determined for order 29 by resonance analysis or any other method. The standard deviations of the other four values correspond to accuracies between 1.2 and 3.5 cm in geoid height. The results are independent of recent comprehensive geoid models and are used to test five such models: the comparisons show that the estimated errors given for the models are quite realistic.

Departmental Reference: Special Systems 8

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1 INTRODUCTION

The rocket which put Cosmos 220 into orbit on 1968 May 7 itself also entered orbit and was designated 1968-40B. Initially it had the following orbital elements[1]: inclination 74.05°; nodal period 99.17 min; apogee and perigee heights, 759 km and 678 km respectively; and eccentricity 0.006.

After some 15 years in orbit, 1968-40B was slowly approaching the condition of 29:2 resonance, *ie* when the satellite makes 29 revolutions while the Earth spins twice, relative to the orbital plane. As a result 1968-40B was, in 1983, placed on the priority list for high-priority observing and remained there for three years, so that the orbit of the satellite could be determined at frequent intervals while it was passing through the resonance. In this Report the orbit of 1968-40B has been determined from radar and optical observations between 1983 January and 1985 July using the RAE orbit determination program PROP in the PROP6 version[2]. The inclination and eccentricity were then analysed over this time to evaluate 29th-order lumped harmonics.

2 THE OBSERVATIONS, ORBITS AND OBSERVATIONAL ACCURACY

2.1 The observations

The orbit of 1968-40B has been determined at 74 epochs, from some 6300 observations, between 1983 January 5 and 1985 July 27. The number of observations used in each determination is given in Table 1 and the source of the observations is also indicated.

There were four groups of observations available. Those listed first in Table 1 were the most accurate, being made by the University of Aston's Hewitt cameras at the Royal Greenwich Observatory site at Herstmonceux and at the Siding Spring Observatory site in Australia: these observations usually have an accuracy of a few seconds of arc. The second consisted of visual observations made by volunteer observers who reported to the University of Aston, and these observations usually have accuracies between 2 and 5 minutes of arc in good observing conditions. The third group is made up of Navspasur observations kindly supplied by the US Naval Research Laboratory, with accuracies of about 2 minutes of arc. The final group, British radar observations, came from the radar trackers at RAF, Fylingdales.

2.2 The orbits

The orbits were determined at approximately 14-day intervals with the aid of the RAE orbit refinement program PROP in the PROP 6 version, and the

Table 1

Sources of the observations used on each orbit

Orbit No.	Source of observation				Total	Orbit No.	Source of observation				Total
	Hewitt camera	Visual	US Navy	British radar			Hewitt camera	Visual	US Navy	British radar	
1		3	70	27	100	38	25H	10	25	21	81
2		11	61	28	100	39	10H	7	18	12	47
3		1	66	33	100	40		13	41	36	90
4		12	61	26	99	41		13	50	31	94
5	12S	2	48	14	76	42			40	31	71
6		5	60	17	82	43	10H		37		47
7	12H	3	12	56	83	44	10H	7	27	6	50
8	6H		53	15	74	45	10H	21	21	17	69
9	10H	8	68	13	99	46	5H	9	23	20	57
10	6H		64	6	76	47		1	36	33	70
11	17H	7	47	29	100	48		2	43	38	83
12		6	47	31	84	49		8	40	34	82
13		1	55	44	100	50		5	54	38	97
14	20H		36	37	93	51	6S	8	55	31	100
15	21H	3	31	26	81	52		24	49	27	100
16	11H	3	26	23	63	53	6S		52	42	100
17	6H	6	35	19	66	54	4S		65	26	95
18	5H	14	25	27	71	55			48	42	90
19		32	42	26	100	56	14S	23	48	15	100
20		10	50	39	99	57	6S	4	48	40	98
21			56	36	92	58		4	45	51	100
22	3H	8	54	30	95	59	6S	3	59	31	99
23	6H	2	36	20	64	60		21	52	27	100
24	6H	6	24	20	56	61	7S		49	43	99
25	6H		31	18	55	62	9S		27	31	67
26		2	40	36	78	63			53	47	100
27		2	39	40	81	64		17	55	28	100
28		5	29	36	70	65	6S	4	62	28	100
29		7	53	40	100	66	4S	1	39	36	80
30	9H	5	36	24	74	67		2	52	45	99
31		6	46	46	98	68	5H	1	65	29	100
32			48	42	90	69		14	53	33	100
33		11	50	39	100	70	12S	9	36	39	96
34		4	47	37	88	71			50	50	100
35			44	29	73	72		4	54	42	100
36	20H		35		55	73	8H	11	35	42	96
37	20H		37	10	67	74	4S	1	43	36	84

S = camera at Siding Spring, Australia. H = camera at Herstmonceux, England

orbital elements at each epoch are listed in Table 2, with the standard deviations below each value. The epoch for each orbit is at 00 hours on the day indicated, and the PROP program fits the mean anomaly M by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 \quad , \qquad (1)$$

where t is the time measured from epoch, and the number of M coefficients used depends on the drag. The satellite 1968-40B was in a nearly circular orbit at a height of about 650 km, where drag is slight: consequently on 31 orbits it was only necessary to use the coefficients M_0 and M_1 ; on 39 orbits the coefficients $M_0 - M_2$ were needed; and on 4 orbits the coefficients $M_0 - M_3$ were required.

The value of ε , the parameter indicating the measure of fit of the observations to the orbit, varied between 0.44 and 1.05, and had an average value of 0.74. For all 74 orbits the standard deviations in inclination were between 0.0005° and 0.0014°, the average being 0.0009° which is equivalent to about 110 m in distance. The values of the standard deviations in eccentricity varied between 0.000001 and 0.000022, and the average value was 0.000011, corresponding to 80 m in distance. The accuracy for the right ascension of the node was nearly the same as that for the inclination.

2.3 The accuracy of the observations

The 3311 US Navy observations used in the orbit determination were of the usual consistent accuracy of about 2 minutes of arc, and those of the British radars were also of their customary standard. The residuals of other observations have been obtained with the aid of the ORES computer program[3] and sent to observers. Table 3 lists the rms residuals of the observing stations with five or more observations accepted in the final orbit determinations.

The residuals of some visual observers are a little higher than usual, but as these rms values include all observations made by each observer, with no attempt to eliminate those made in bad conditions of visibility, this is not surprising.

The Hewitt cameras did very well: the Herstmonceux camera contributed 265 observations with rms residual of 8 seconds of arc, equivalent to about 30 m in position, and the Siding Spring camera contributed 81 observations with rms residual of 3 seconds of arc, equivalent to about 10 m. Many of the 74 orbits

Table 2

Values of orbital parameters at 74 epochs with standard deviations

	MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	M_3	ε	N	D
1	45339	1983 Jan 5	7067.0870	0.005515	74.0474	28.877	39.9	243.858	5262.0693	0.0036		0.55	77	7.9
			2	11	12	1	1		3	1				
2	45351	1983 Jan 17	7066.9854	0.005118	74.0476	5.931	15.9	3.338	5262.1828	0.0040		0.70	86	7.9
			3	16	11	1	1		3	2				
3	45366	1983 Feb 1	7066.8993	0.004618	74.0522	337.247	343.2	64.240	5262.2792	0.0038		0.57	84	7.6
			2	14	12	1	1		3	1				
4	45386	1983 Feb 21	7066.7927	0.004064	74.0497	299.006	293.8	147.995	5262.3982	0.0015		0.51	89	8.4
			2	8	9	1	2		2	1				
5	45401	1983 Mar 8	7066.7328	0.004095	74.0459	270.318	258.9	211.848	5262.4649	0.0013		1.05	75	7.1
			4	15	12	1	2		4	1				
6	45418	1983 Mar 25	7066.6701	0.004381	74.0448	237.805	211.0	357.415	5262.5349	0.0033		0.44	65	9.5
			2	10	8	1	1		2	1				
7	45430	1983 Apr 6	7066.6081	0.004786	74.0477	214.857	183.6	122.248	5262.6044	0.0015		0.47	73	7.5
			1	11	6	1	1		1	1				
8	45438	1983 Apr 14	7066.5794	0.005067	74.0497	199.558	166.5	85.849	5262.6366	0.0026		0.46	58	7.5
			3	9	6	1	1		3	1				
9	45455	1983 May 1	7066.4942	0.005597	74.0491	167.049	133.2	234.604	5262.7318	0.0025	0.00016	0.56	90	9.7
			3	7	5	1	<1		3	1	3			
10	45480	1983 May 26	7066.3749	0.005879	74.0450	119.231	87.0	350.230	5262.8649	0.0035		0.54	74	9.9
			2	6	7	1	1		2	1				
11	45496	1983 Jun 11	7066.3070	0.005717	74.0496	88.629	57.6	282.015	5262.9410	0.0022	0.00029	0.57	99	7.5
			5	8	6	1	1		6	1	6			
12	45508	1983 Jun 23	7066.2519	0.005449	74.0486	65.675	34.9	51.656	5263.0025	0.0047	0.00057	0.77	77	7.6
			10	18	12	1	1		11	2	11			
13	45527	1983 Jul 12	7066.2061	0.004838	74.0459	29.332	355.4	288.039	5263.0535	0.0009		0.69	98	7.9
			2	12	8	1	1		2	1				
14	45550	1983 Aug 4	7066.1584	0.004214	74.0482	345.327	300.1	328.944	5263.1069	0.0008	0.00028	0.74	91	5.7
			1	9	8	1	1		1	2	8			
15	45560	1983 Aug 14	7066.1272	0.004092	74.0531	326.203	273.3	18.466	5263.1420			0.89	81	3.8
			1	10	9	1	2		1					
16	45564	1983 Aug 18	7066.1229	0.004075	74.0502	318.547	262.0	182.379	5263.1468			0.87	61	3.7
			1	8	11	2	2		1					
17	45573	1983 Aug 27	7066.0888	0.004182	74.0506	301.333	239.5	11.371	5263.1849			0.66	63	4.0
			7	10	8	1	2		8					
18	45577	1983 Aug 31	7066.0803	0.004267	74.0492	293.678	228.9	175.454	5263.1943			0.82	68	3.8
			8	12	10	2	2		9					
19	45584	1983 Sep 7	7066.0630	0.004532	74.0466	280.289	211.5	282.683	5263.2135	0.0016		0.80	99	5.0
			5	16	10	1	2		6	4				
20	45595	1983 Sep 18	7066.0336	0.004938	74.0436	259.240	186.4	194.332	5263.2462	0.0019		0.82	99	6.0
			4	19	10	1	1		4	3				
21	45610	1983 Oct 3	7065.9964	0.005404	74.0460	230.543	154.9	270.777	5263.2880	0.0017		0.67	89	6.9
			3	14	9	1	1		3	2				
22	45635	1983 Oct 28	7065.9138	0.005956	74.0468	182,716	107.3	39.977	5263.3803	0.0010		0.81	95	5.7
			4	7	7	1	2		5	3				
23	45642	1983 Nov 4	7065.8915	0.005982	74.0479	169.326	94.3	148.519	5263.4053	0.0026		0.83	64	4.0
			8	10	7	1	2		9	7				
24	45646	1983 Nov 8	7065.8834	0.005985	74.0462	161.672	86.8	313.466	5263.4142	0.0045		0.69	55	3.7
			7	9	6	1	1		7	7				
25	45650	1983 Nov 12	7065.8670	0.005974	74.0464	154.020	78.9	118.470	5263.4326	0.0050		0.65	55	4.0
			7	8	7	1	2		8	4				
26	45662	1983 Nov 24	7065.8432	0.005859	74.0426	131.058	57.2	253.805	5263.4590			0.83	78	5.8
			4	14	11	1	2		5					
27	45677	1983 Dec 9	7065.8172	0.005430	74.0461	102.358	28.6	333.335	5263.4883	0.0012		0.93	79	5.8
			5	21	13	2	2		6	4				
28	45697	1983 Dec 29	7065.7800	0.004675	74.0451	64.089	345.4	80.175	5263.5298			0.96	70	5.6
			6	22	14	2	2		7					
29	45705	1984 Jan 6	7065.7664	0.004469	74.0462	48.783	326.8	51.101	5263.5450	0.0008		0.83	97	6.3
			4	13	10	1	1		4	2				
30	45719	1984 Jan 20	7065.7425	0.004236	74.0441	21.992	291.2	270.537	5263.5717			0.77	70	6.0
			3	10	9	1	3		3					
31	45730	1984 Jan 31	7065.7154	0.004147	74.0444	0.942	261.8	186.090	5263.6019	0.0010		0.71	97	6.6
			4	9	9	1	2		5	2				
32	45748	1984 Feb 18	7065.6665	0.004498	74.0470	326.500	215.7	212.329	5263.6567	0.0013		0.82	90	7.7
			3	16	11	1	2		4	2				
33	45764	1984 Mar 5	7065.6115	0.005026	74.0448	295.884	179.5	156.558	5263.7181	0.0022		0.85	98	6.8
			3	20	10	1	1		4	2				
34	45774	1984 Mar 15	7065.5813	0.005341	74.0409	276.748	158.6	212.213	5263.7516	0.0022		0.76	84	5.7
			4	9	10	1	1		5	3				
35	45795	1984 Apr 5	7065.4531	0.005867	74.0426	236.553	117.9	186.768	5263.8951	0.0041		0.65	70	7.5
			3	11	10	1	1		4	2				
36	45803	1984 Apr 13	7065.4144	0.005946	74.0472	221.245	103.3	160.781	5263.9385			0.68	54	5.5
			1	8	9	1	1		1					
37	45812	1984 Apr 22	7065.3851	0.005941	74.0496	204.024	86.5	356.832	5263.9714	0.0019		0.69	66	5.5
			<1	6	7	1	1		1	1				

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Table 2 (concluded)

	MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	M_3	ε	N	D
38	45817	1984 Apr 27	7065.3669	0.005920	74.0480	194.453	77.5	25.887	5263.9917	0.0025		0.92	79	4.0
			1	6	8	1	2		1	1				
39	45821	1984 May 1	7065.3534	0.005888	74.0463	186.798	70.0	193.207	5264.0068			0.84	47	3.5
			1	5	13	2	2		1					
40	45826	1984 May 6	7065.3397	0.005820	74.0426	177.232	61.5	222.425	5264.0218	0.0014		0.90	89	5.8
			5	13	12	2	2		5	3				
41	45834	1984 May 14	7065.3224	0.005625	74.0436	161.924	46.2	197.304	5264.0412	0.0014		0.62	92	5.8
			3	11	8	1	1		3	2				
42	45853	1984 Jun 2	7065.2512	0.004985	74.0409	125.558	7.9	93.605	5264.1207	0.0016		0.69	70	5.7
			5	18	10	1	1		5	3				
43	45861	1984 Jun 10	7065.2279	0.004761	74.0463	110.245	350.0	69.309	5264.1470	0.0022		0.64	47	5.6
			5	8	10	1	1		5	2				
44	45866	1984 Jun 15	7065.2162	0.004572	74.0469	100.680	338.2	99.217	5264.1600			0.72	49	3.7
			2	8	11	2	1		2					
45	45870	1984 Jun 19	7065.2083	0.004411	74.0465	93.023	328.7	267.192	5264.1689			0.80	66	3.7
			1	17	9	1	2		1					
46	45878	1984 Jun 27	7065.1916	0.004262	74.0477	77.712	309.4	243.255	5264.1877			0.73	53	3.0
			6	11	13	1	2		7					
47	45897	1984 Jul 16	7065.1647	0.004077	74.0422	41.343	259.2	141.867	5264.2175			0.87	70	5.2
			5	12	13	2	3		6					
48	45906	1984 Jul 25	7065.1542	0.004192	74.0415	24.116	236.0	340.336	5264.2291			0.87	83	5.9
			4	13	11	1	2		5					
49	45934	1984 Aug 22	7065.1208	0.005170	74.0489	330.524	170.6	78.463	5264.2669			0.73	79	5.3
			4	19	10	1	1		4					
50	45947	1984 Sep 4	7065.1034	0.005569	74.0487	305.644	144.5	85.825	5264.2863			0.73	94	5.8
			3	14	9	1	1		4					
51	45955	1984 Sep 12	7065.0897	0.005768	74.0445	290.332	129.3	62.806	5264.3015	0.0007		0.59	98	5.8
			3	6	6	1	<1		4	2				
52	45964	1984 Sep 21	7065.0783	0.005916	74.0426	273.106	112.8	262.030	5264.3141	0.0014		0.58	96	6.2
			3	8	7	1	1		3	2				
53	45980	1984 Oct 7	7065.0527	0.005940	74.0446	242.473	82.8	216.544	5264.3428	0.0010		0.60	98	6.0
			3	6	6	1	<1		3	2				
54	45986	1984 Oct 13	7065.0438	0.005878	74.0482	230.989	71.8	109.605	5264.3529	0.0008		0.60	89	5.6
			2	4	6	1	<1		3	2				
55	46002	1984 Oct 29	7065.0184	0.005571	74.0486	200.369	41.9	64.741	5264.3814	0.0008		0.73	87	5.6
			4	14	9	1	1		5	3				
56	46017	1984 Nov 13	7065.0017	0.005091	74.0460	171.662	11.5	158.028	5264.3999			0.59	97	5.2
			<1	1	6	<1	<1		1					
57	46038	1984 Dec 4	7064.9680	0.004338	74.0415	131.464	322.7	145.165	5264.4374			0.85	98	5.8
			4	17	9	1	1		4					
58	46056	1984 Dec 22	7064.9546	0.004092	74.0504	97.011	276.6	186.080	5264.4527	0.0003		0.80	99	8.3
			3	10	10	1	2		3	1				
59	46077	1985 Jan 12	7064.9318	0.004434	74.0435	56.811	222.4	174.259	5264.4779			0.86	99	6.5
			4	6	10	1	1		5					
60	46091	1985 Jan 26	7064.9176	0.004829	74.0463	30.014	189.4	46.650	5264.4939	0.0004		0.57	96	6.3
			2	14	7	1	1		3	2				
61	46104	1985 Feb 8	7064.9057	0.005270	74.0445	5.123	162.0	56.926	5264.5072	0.0008		0.89	93	5.9
			2	5	5	1	1		2	3				
62	46109	1985 Feb 13	7064.9007	0.005423	74.0473	355.554	151.8	88.619	5264.5129			0.86	67	4.0
			2	6	10	1	1		2					
63	46121	1985 Feb 25	7064.8919	0.005761	74.0489	332.591	128.4	236.776	5264.5229			0.90	97	4.6
			5	15	11	1	2		6					
64	46134	1985 Mar 10	7064.8821	0.005964	74.0468	307.710	104.6	247.432	5264.5337			0.89	97	5.2
			4	10	10	1	2		4					
65	46144	1985 Mar 20	7064.8784	0.005998	74.0431	288.567	85.6	311.077	5264.4376			0.63	90	5.9
			3	5	8	1	<1		3					
66	46154	1985 Mar 30	7064.8742	0.005865	74.0417	269.417	67.9	14.763	5264.5422			0.93	78	6.0
			5	6	11	1	1		6					
67	46170	1985 Apr 15	7064.8711	0.005541	74.0418	238.788	37.5	332.714	5264.5457			0.81	96	7.5
			2	15	10	1	1		3					
68	46182	1985 Apr 27	7064.8613	0.005208	74.0484	215.822	12.8	121.249	5264.5570			0.81	95	6.0
			3	15	8	1	1		4					
69	46195	1985 May 10	7064.8554	0.004703	74.0464	190.939	344.9	132.293	5264.5634			0.56	95	7.0
			2	13	7	1	1		3					
70	46205	1985 May 20	7064.8464	0.004390	74.0462	171.795	320.1	196.257	5264.5735	0.0012		0.85	96	6.9
			2	6	8	1	1		2	1				
71	46221	1985 Jun 5	7064.8382	0.004106	74.0433	141.164	279.2	154.757	5264.5825			0.84	100	7.5
			3	9	10	1	3		4					
72	46234	1985 Jun 18	7064.8289	0.004175	74.0450	116.276	244.3	166.178	5264.5930			0.88	98	7.4
			3	14	11	1	2		4					
73	46251	1985 Jul 5	7064.8232	0.004576	74.0489	83.734	202.7	347.410	5264.5996			0.88	96	6.4
			3	18	8	1	1		4					
74	46273	1985 Jul 27	7064.8101	0.005325	74.0451	41.614	154.8	201.001	5264.6141			0.93	84	5.3
			5	5	12	1	1		6					

KEY
- MJD modified Julian day
- a semi major axis (km)
- e eccentricity
- i inclination (deg)
- Ω right ascension of ascending node (deg)
- ω argument of perigee (deg)
- M_0 mean anomaly at epoch (deg)
- M_1 mean motion n (deg/day)
- M_2, M_3 later coefficients in the polynomial for M
- ε measure of fit
- N number of observations accepted in each orbit determination
- D time covered by the observations (days)

Table 3

Residuals for observing stations with five or more observations accepted in the final orbit determination

Station		Number of observations		Rms residuals Minutes of arc		
		Accepted	Rejected	RA	Dec	Total
414	Capetown	20	1	1.7	2.1	2.7
1184	Eilenburg	7	0	5.9	1.9	6.2
2265	Farnham	24	3	5.4	5.0	7.4
2392	Cowbeech	8	1	1.0	2.4	2.6
2414	Bournemouth	132	0	3.2	3.5	4.7
2418	Sunningdale	27	0	2.3	2.2	3.2
2420	Willowbrae	83	6	4.0	3.4	5.2
2430	Stevenage 4	8	0	1.7	2.1	2.7
2437	Warrington	5	0	6.1	7.4	9.6
2539	Dymchurch	25	0	1.5	1.6	2.2
2657	Bridgwater	30	0	1.8	2.2	2.8
2658	Hillsborough	8	0	3.0	1.6	3.4
2659	Herstmonceux 3 (Hewitt camera)	265	2	0.10	0.10	0.14
2665	Cluj 3	5	0	4.3	5.9	7.3
4156	Apeldoorn	7	0	1.2	3.8	4.0
9652	Siding Spring (Hewitt camera)	81	4	0.03	0.04	0.05

utilize more than one Hewitt camera transit and the PROP orbital model is not accurate to better than 50 m over an interval of more than a few days because lunisolar perturbations are neglected. So orbital error almost certainly contributes to the Hewitt camera residuals: the cameras are more accurate than the residuals indicate. The Herstmonceux camera suffered more from this effect, because many of the orbit determinations included multiple transits: for example, orbit 38 used four Hewitt camera plates at daily intervals.

3 EQUATIONS FOR 29:2 RESONANCE

The theory for 29:2 resonance is detailed in Ref 4, where all the parameters used are defined. The theoretical equation for the change in inclination at resonance, for orbits with small eccentricity, is

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{29}\Bigg[\frac{R}{a}(29 - 2\cos i)\bar{F}_{30,29,14}\left\{\bar{S}_{29}^{0,2}\sin\Phi + \bar{C}_{29}^{0,2}\cos\Phi\right\}$$

$$+ 16e(29 - \cos i)\bar{F}_{29,29,14}\left\{\bar{C}_{29}^{1,1}\sin(\Phi - \omega) - \bar{S}_{29}^{1,1}\cos(\Phi - \omega)\right\}$$

$$+ 12e(29 - 3\cos i)\bar{F}_{29,29,13}\left\{\bar{C}_{29}^{-1,3}\sin(\Phi + \omega) - \bar{S}_{29}^{-1,3}\cos(\Phi + \omega)\right\}$$

$$+ \text{terms in } e^{|q|}\,{\cos \atop \sin}(\gamma\Phi - q\omega)\Bigg], \qquad \ldots\ldots(2)$$

where $$\Phi = 2(\omega + M) + 29(\Omega - \nu) \qquad (3)$$

is the resonance angle, ω being the argument of perigee, M the mean anomaly, Ω the right ascension of the node and ν the sidereal angle. The quantities γ and q are integers, but only three terms are given explicitly in equation (2) because it is believed the others are small: terms with $q = \pm 2$ have e^2 as a multiplying factor and, for 1968-40B, e is 0.006 at most during the time of 29:2 resonance; while the terms with $\gamma = 2$ are associated with harmonics of order 58, which should be much smaller than those of order 29. The $\bar{F}$ in equation (2) are functions of inclination only, defined in Ref 4.

The theoretical equation for the variation of e, for small e, is[4]

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{29}\Bigg[\!\!\Bigg[-\frac{R}{a}\bar{F}_{30,29,14}e\left(\bar{S}_{29}^{0,2}\sin\Phi + \bar{C}_{29}^{0,2}\cos\Phi\right)$$

$$- 16\bar{F}_{29,29,14}\left\{\bar{C}_{29}^{1,1}\sin(\Phi - \omega) - \bar{S}_{29}^{1,1}\cos(\Phi - \omega)\right\}$$

$$+ 12\bar{F}_{29,29,13}\left\{\bar{C}_{29}^{-1,3}\sin(\Phi + \omega) - \bar{S}_{29}^{-1,3}\cos(\Phi + \omega)\right\}$$

$$+ \text{terms in } \left[e^{|q|-1}\left\{q - (\gamma + q)e^2\right\}{\cos \atop \sin}(\gamma\Phi - q\omega)\right]\Bigg]\!\!\Bigg] . \qquad (4)$$

The main terms in equation (4) are expected to be those with $(\gamma,q) = (1,1)$ and $(1,-1)$, but the $(\gamma,q) = (1,0)$ term is included so that the analysis conforms with that for inclination.

The quantities such as $\bar{S}_{29}^{0,2}$ in equations (2) and (4) are lumped geopotential harmonics of order 29, which can be expressed as linear sums of individual coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$,

$$\bar{C}_m^{q,k} = \sum_{\ell} Q_\ell^{q,k} \bar{C}_{\ell m} \quad \text{and} \quad \bar{S}_m^{q,k} = \sum_{\ell} Q_\ell^{q,k} \bar{S}_{\ell m} \quad , \tag{5}$$

and these individual coefficients can be evaluated when enough lumped harmonics from satellites at different inclinations are available. The summation for ℓ in equation (5) is in steps of 2, beginning at the lowest ℓ, which is either 29 or 30 here (see Ref 4). Also $k = 2\gamma - q$ for 29:2 resonance.

The orders of magnitude of the terms in the equations for di/dt and de/dt can be estimated, as the $\bar{C}_{\ell m}$ (or $\bar{S}_{\ell m}$) are expected to be of order $10^{-5}/\ell^2$, so the value of $\bar{C}_m^{q,k}$ (or $\bar{S}_m^{q,k}$) can be taken to be of order $\left\{\sum(Q_\ell \times 10^{-5}/\ell^2)^2\right\}^{\frac{1}{2}}$, the Q coefficients being obtained for specific values of (γ,q) using the RAE computer program PROF.

Estimating the orders of magnitude of the terms in equation (2) for inclination suggests that, if just the $(\gamma,q) = (1,0)$ term is used, the likely errors are 2% from the neglect of the (2,0) term and 8% from the neglect of the $(1,\pm1)$ terms. In equation (4) the $(\gamma,q) = (1,0)$ term only contributes about 0.5%. As mentioned before, the $q = 2$ terms contain a factor e^2 and therefore they will not be needed.

4 ANALYSIS OF INCLINATION AT 29:2 RESONANCE

Cosmos 220 rocket passed through exact 29:2 resonance on 1984 Aug 15 and its orbit has been determined either side of resonance, with the rate of change of resonance angle, $\dot{\Phi}$, increasing from -4.3 to +0.7 deg/day. The variation of both $\dot{\Phi}$ and the resonance angle Φ, given by equation (3), are shown in Fig 1. The decrease of $\dot{\Phi}$ became very slow in the last 18 months due to the reduction in solar activity.

Before the changes in inclination due to resonance can be analysed, all other known perturbations must be removed. The 74 values of inclination in Table 2 were therefore cleared of lunisolar and zonal harmonic perturbations, by using the computer program PROD[5] with 1-day integration steps, and the perturbation due to the $J_{2,2}$ tesseral harmonic, which is recorded on each PROP run, was also removed from each value of inclination. Perturbations due to earth and ocean tides should not exceed 50 m and need not be considered, since the values of inclination have an average accuracy of 110 m, the most accurate being 0.0005° which is approximately equivalent to 65 m.

The values of inclination, cleared of the perturbations mentioned, and with the standard deviations quoted in Table 2, were fitted with equation (2) in integrated form using the RAE THROE[6] computer program. This program removes the perturbations due to atmospheric rotation and lunisolar precession of the Earth's axis before fitting the remaining perturbation due to resonance. The density scale height, H, was taken as 85 km, appropriate to a height of 683 km, 0.4H above perigee[7], and the atmospheric rotation rate, Λ, was taken[8] as 0.9 rev/day. The values of M_2 were altered to mean values, $\bar{M}_2$, by the technique described in Ref 9.

The first fitting of the values of inclination by THROE, with $(\gamma,q) = (1,0)$, gave $\varepsilon = 2.29$, where ε is the measure-of-fit parameter. On inspection of the PROP runs for some of the ill-fitting values of inclination, it was found that the runs included quite a few low-elevation radar observations, and relaxing the standard deviations of the range values had a beneficial effect on the values of inclination. After these relaxations the value of ε on the next THROE run was 1.081 and the values of the lumped harmonics were as follows:

$$10^9\bar{C}_{29}^{0,2} = -2.6 \pm 0.5 \quad , \qquad 10^9\bar{S}_{29}^{0,2} = 8.9 \pm 1.0 \ . \tag{6}$$

For this fitting, one value of inclination had its standard deviation increased by a factor of 1.5 to ensure that all weighted residuals were less than 2ε.

The addition of the $(\gamma,q) = (1,\pm 1)$ terms did not improve the standard deviation of $(\bar{C},\bar{S})_{29}^{0,2}$ and the additional harmonics were not well determined. The values obtained from the THROE run were:

$$\left.\begin{array}{llll} \bar{C}_{29}^{0,2} = -2.2 \pm 0.5 & & \bar{S}_{29}^{0,2} = 9.2 \pm 1.0 \\ \bar{C}_{29}^{1,1} = 15 \pm 13 & & \bar{S}_{29}^{1,1} = 62 \pm 24 \\ \bar{C}_{29}^{-1,3} = -94 \pm 36 & & \bar{S}_{29}^{-1,3} = -24 \pm 37 \end{array}\right\} \tag{7}$$

with $\varepsilon = 0.927$. The values of $(\bar{C},\bar{S})_{29}^{1,1}$ and $(\bar{C},\bar{S})_{29}^{-1,3}$ were of the right order according to the $10^{-5}/\ell^2$ rule, but they were poorly determined. The results are given because the run was needed for the simultaneous fitting of inclination and eccentricity. The addition of the $(\gamma,q) = (2,0)$ terms also proved unhelpful.

The values of inclination, cleared of all known perturbations except those due to resonance, are plotted in Fig 2. The theoretical curve derived from the THROE fitting with $(\gamma,q) = (1,0)$, that gave the values in equation (6), is also shown, as a full line.

5 ANALYSIS OF ECCENTRICITY AT 29:2 RESONANCE

Before the changes in eccentricity due to resonance can be analysed, the values of eccentricity in Table 2 have first to be cleared of perturbations due to zonal harmonics, atmospheric drag and lunisolar perturbations.

The removal of zonal harmonic and lunisolar perturbations was performed by using the PROD[5] computer program, and the variation due to drag in an atmosphere with day-to-night variation was calculated using the theory given in the Appendix of Ref 10 with scale height H taken as 106 km, appropriate to a mean height of 734 km. This theory assumes that the atmospheric density depends on the geocentric angular distance from the point of maximum density, which has been taken as 14 h local time[11].

In Fig 3a the values of eccentricity from Table 2 are given as circles, and these values after removal of zonal harmonic and lunisolar perturbations are indicated by triangles. The correction Δe_D for drag in an atmosphere with day-to-night variation is shown in Fig 3b. The values of eccentricity cleared of these perturbations are now ready to be fitted with equation (4) in integrated form, using THROE.

After the values of M_2 had been altered to mean values, $\bar{M}_2$, as for the inclination, the first THROE fitting with $(\gamma,q) = (1,\pm 1)$ was performed and the value of ε obtained was 3.91. However, two of the eccentricity values, had standard deviations less than 0.000005; so both were increased to that value, in case there was a significant effect from the neglect of earth and ocean tides. Six values of eccentricity had to have their standard deviations increased by a factor of 2, and two by a factor of 4, to keep all the weighted residuals less than 2ε. The THROE run with these adjustments gave $\varepsilon = 2.53$.

In some previous fittings of values of eccentricity with THROE, it was found that adjustment of the odd zonal harmonics was needed. The same method has been used here as in Ref 12, *ie* to add an increment ΔJ_3 to the J_3 value used in the PROD model. The lowest value of ε, 2.497, was obtained with $\Delta J_3 = -0.015 \times 10^{-6}$.

This final fitting of the values with $(\gamma,q) = (1,\pm1)$ and with $\Delta J_3 = -0.015 \times 10^{-6}$, gave values of the lumped harmonics as follows:

$$\left.\begin{array}{llll} 10^9\bar{C}_{29}^{1,1} = 53.4 \pm 3.8 & & 10^9\bar{S}_{29}^{1,1} = -10.3 \pm 5.6 \\ 10^9\bar{C}_{29}^{-1,3} = 18.9 \pm 6.9 & & 10^9\bar{S}_{29}^{-1,3} = -3.9 \pm 7.8 \end{array}\right\} \qquad (8)$$

The values of eccentricity, cleared of all known perturbations larger than the standard deviations of the values (except those due to resonance), are plotted in Fig 4. The fitted theoretical curve is that of the THROE run which produced the values in equation (8), and is shown as a full line.

6 INCLINATION AND ECCENTRICITY FITTED SIMULTANEOUSLY

The values of inclination and eccentricity fitted separately by THROE can be fitted simultaneously using the RAE computer program SIMRES developed by Dr R.H. Gooding. This program combines the results from a number of THROE runs (each with the same set of (γ,q) terms), and produces a single set of coefficients to fit the data. For this SIMRES fitting, the results of THROE runs with $(\gamma,q) = (1,0)(1,1)$ and $(1,-1)$ were used. The SIMRES program allows a choice of weighting, so that the contributing THROE runs can be given more or less weight according to their accuracy of fit, which is indicated by the value of ε.

The THROE fitting of inclination with $(\gamma,q) = (1,0)(1,1)$ and $(1,-1)$ gave $\varepsilon = 0.927$, and for eccentricity the value of ε was 2.100, when fitted with the same terms. For the SIMRES fitting, therefore, the weighting of e was downgraded by a factor equal to the ratio of the values of ε on the THROE fittings, namely 2.265 (=2.100/0.927). The values of the lumped harmonics given by this SIMRES fitting are:

$$\left.\begin{array}{llll} 10^9\bar{C}_{29}^{0,2} = -2.7 \pm 0.5 & & 10^9\bar{S}_{29}^{0,2} = 8.2 \pm 1.0 \\ 10^9\bar{C}_{29}^{1,1} = 51.2 \pm 3.5 & & 10^9\bar{S}_{29}^{1,1} = -7.5 \pm 5.3 \\ 10^9\bar{C}_{29}^{-1,3} = 16.1 \pm 6.5 & & 10^9\bar{S}_{29}^{-1,3} = -2.3 \pm 7.3 \end{array}\right\} . \qquad (9)$$

The SIMRES fittings of inclination and eccentricity are shown pictorially in Figs 2 and 4, as broken lines.

The lumped harmonics obtained from the fitting of inclination and eccentricity separately, given in equations (6) and (8), are very similar to those obtained from the simultaneous fitting, given in equation (9): all like coefficients are within one standard deviation of each other. So either set of values could be used in future determinations of the individual 29th-order harmonic coefficients in the geopotential. The SIMRES values are probably better, for two reasons. First, they take into account the $(\gamma,q) = (1,\pm 1)$ terms for inclination, and these may be significant because, when estimating the orders of magnitude of the terms (see section 3), it was found that neglect of the $(1,\pm 1)$ terms in fitting the inclination could give an error of 8%. Second, the SIMRES curve does seem to provide the better fit in Fig 2, especially between MJD 45650 and 45900, and ε is slightly lower (1.05 as against 1.08).

7 EQUATIONS FOR INDIVIDUAL COEFFICIENTS

The lumped harmonics in equation (9) can be expressed as linear sums of 29th-order individual coefficients: see equation (5). The Q coefficients in these equations depend[4] on the ratios of the eccentricity functions $G_{\ell pq}$, and can be evaluated using the RAE computer program PROF. However, a correction factor has to be applied to the values of Q, as it was assumed in the PROF program that the e^2 terms in the functions $G_{\ell pq}$ could be neglected (see Ref 4). This leads to large errors in Q if e is large; but here, with e approximately 0.005, the correction is almost negligible, being less than 1%. The correction has, nevertheless, been made. The values of the lumped harmonics from THROE and SIMRES also need a very small correction, which has been made in equations (6) to (9).

The equations given below for the individual coefficients have been terminated when the expected contribution from the coefficients permanently falls to less than 5% of the largest contribution.

The resulting six equations are as follows:

$$\bar{C}_{29}^{0,2} = \bar{C}_{30,29} + 0.044\bar{C}_{32,29} - 0.356\bar{C}_{34,29} - 0.349\bar{C}_{36,29} - 0.175\bar{C}_{38,29} - 0.000\bar{C}_{40,29}$$

$$+ 0.101\bar{C}_{42,29} + 0.122\bar{C}_{44,29} \quad , \qquad (10)$$

$\bar{S}_{29}^{0,2}$: the equation is the same as (10) with S instead of C , (11)

$$\bar{C}_{29}^{1,1} = \bar{C}_{29,29} - 1.576\bar{C}_{31,29} - 0.726\bar{C}_{33,29} + 0.244\bar{C}_{35,29} + 0.635\bar{C}_{37,29} + 0.524\bar{C}_{39,29}$$

$$+ 0.198\bar{C}_{41,29} - 0.099\bar{C}_{43,29} - 0.249\bar{C}_{45,29} - 0.246\bar{C}_{47,29} - 0.148\bar{C}_{49,29} , \quad ......(12)$$

$\bar{S}_{29}^{1,1}$: the equation is the same as (12) with S instead of C , (13)

$$\bar{C}_{29}^{-1,3} = \bar{C}_{29,29} - 0.590\bar{C}_{31,29} - 0.759\bar{C}_{33,29} - 0.464\bar{C}_{35,29} - 0.087\bar{C}_{37,29} + 0.183\bar{C}_{39,29}$$

$$+ 0.290\bar{C}_{41,29} + 0.258\bar{C}_{43,29} + 0.148\bar{C}_{45,29} , \quad (14)$$

$\bar{S}_{29}^{-1,3}$: the equation is the same as (14) with S instead of C . (15)

8 LUMPED HARMONICS FROM 1968-40B COMPARED WITH THOSE FROM COMPREHENSIVE GEOID MODELS

It is interesting to compare the values of lumped harmonics obtained here with those from comprehensive gravity-field models. Five models have been used for comparison: GEM 10B and GEM 10C[13], the model produced in 1981 by R.H. Rapp[14], GRIM 3-L1[15] and GEM-T1[16]. The values of 29th-order lumped harmonics given by substituting the values of the individual coefficients from the models into equations (10) to (15) appear in Table 4.

The Goddard Earth Model GEM 10B extends to order and degree 36, and GEM 10C consists of the GEM 10B solution up to degree 36, together with some 31000 coefficients of order and degree up to 180, derived from analysis of altimeter measurements over the oceans. Equations (10) to (15) require coefficients above 36, some as high as degree 49, but the lumped harmonics in Table 4 for GEM 10B are truncated at degree 36. The expected accuracy of the GEM 10B coefficients has been assumed[17] to be 3×10^{-9} . The GEM 10C accuracy (above degree 36) is certainly poorer[18] and is rather arbitrarily taken as 5×10^{-9} . The standard deviations for the GEM 10B and GEM 10C values in Table 4 have been assessed using these accuracies for the individual coefficients.

The comprehensive geopotential model produced by R.H. Rapp at the Ohio State University in 1981 gives the individual coefficients to order and degree 180, and is derived from Seasat altimeter data, terrestrial gravity measurements and other data. An accuracy estimate for each coefficient is also given, so the standard deviations for the lumped harmonics in Table 4 have been assessed using

these accuracies. Finally, the lumped harmonics obtained by using the individual coefficients from GRIM3-L1 and GEM-T1 are given in Table 4. These models only extend to degree and order 36, so the equations (10) to (15) have to be truncated at this value, as with GEM10B. The standard deviations are calculated from the accuracies given for the individual coefficients in both cases.

Table 4

Values of lumped harmonics from 1968-40B and comprehensive geoid models

	$10^9\bar{C}_{29}^{0,2}$	$10^9\bar{S}_{29}^{0,2}$	$10^9\bar{C}_{29}^{1,1}$	$10^9\bar{S}_{29}^{1,1}$	$10^9\bar{C}_{29}^{-1,3}$	$10^9\bar{S}_{29}^{-1,3}$
1968-40B	2.7 ± 0.5	8.2 ± 1.0	51.2 ± 3.5	-7.5 ± 5.3	16.1 ± 6.5	-2.3 ± 7.3
GEM 10C	-0.5 ± 3	5.2 ± 3	25.9 ± 8	-12.5 ± 8	15.3 ± 5	-5.2 ± 5
GEM 10B	1.3 ± 3	4.9 ± 3	24.7 ± 6	-8.9 ± 6	14.4 ± 4	-5.4 ± 4
Rapp (1981)	-0.5 ± 1.9	7.1 ± 2.5	32.0 ± 4.2	-1.2 ± 6.5	13.3 ± 3.1	-11.7 ± 4.3
GRIM3-L1	0.8 ± 3.3	5.8 ± 3.3	34.5 ± 6.0	-10.3 ± 6.0	24.6 ± 4.3	-13.6 ± 4.3
GEM-T1	6.5 ± 6.2	2.7 ± 6.2	34.7 ±10.4	11.6 ± 10.4	24.2 ± 8.8	8.0 ± 8.8

On examination of Table 4 it is seen that all the lumped harmonics from the models, except $\bar{C}_{29}^{-1,1}$, agree well with those from 1968-40B, within 0.6 times the sum of their standard deviations on average; only the Rapp value for $\bar{C}_{29}^{0,2}$ and the GEM-T1 value for $\bar{S}_{29}^{-1,1}$ differ from the 1968-40B values by more than the sum of their standard deviations. The values for $\bar{C}_{29}^{-1,1}$ do not agree so well with that from 1968-40B: on average the difference is 2.1 times the sum of the standard deviations. The value of $\bar{C}_{29}^{-1,1}$ from 1968-40B is large and well-defined, originating from the large and well-defined increase in eccentricity between MJD 45400 and 45700 in Fig 4.

The standard deviations of the values from 1968-40B are much better than those of the models for the first pair of coefficients, somewhat better for the second pair, and rather worse for the third pair.

The good agreement between the lumped harmonics in Table 4 is very satisfying, as it shows that the 29th-order coefficients in the models must be of the right magnitude and that their estimated errors are quite realistic. A previous analysis[19] of the 29:2 resonance of 1967-104B with less accurate orbits led to similar though less definite conclusions. Hopefully in the future more satellites can be analysed at other inclinations as they pass through 29:2 resonance, and then the individual coefficients can be evaluated from satellite orbit analysis.

In conclusion, it is worth emphasizing that the results obtained here from 1968-40B are completely independent of the models: the gravity field used in PROP is from a much earlier model, of zonal harmonics and $J_{2,2}$ only; and 1968-40B is not among the satellites utilized in the models of Table 4.

9 APPROXIMATE ACCURACY IN GEOID HEIGHT

Equations (10) to (15) are useful in allowing an approximate assessment of the accuracy of the lumped harmonics evaluated for 1968-40B, in terms of an accuracy in geoid height. The accuracy σ_g in geoid height may be estimated approximately as $R\sigma/Q^*$, where σ is the error in the lumped harmonic and

$$Q^* = \left\{\sum\left(\frac{Q_\ell^{q,k}\ell_0^2}{\ell^2}\right)^2\right\}^{\frac{1}{2}} .$$

The values of σ_g for each lumped harmonic are as follows:

	$\bar{C}_{29}^{0,2}$	$\bar{S}_{29}^{0,2}$	$\bar{C}_{29}^{1,1}$	$\bar{S}_{29}^{1,1}$	$\bar{C}_{29}^{-1,3}$	$\bar{S}_{29}^{-1,3}$
σ_g (cm)	0.3	0.6	1.2	1.8	3.1	3.5

The values for $(q,k) = (1,1)$ are more accurate than those for $(q,k) = (-1,3)$, probably because the orbit determination covered the $(\gamma,q) = (1,1)$ resonance $(\dot{\Phi} = \dot{\omega})$, but did not continue long enough to cover fully the $(\gamma,q) = (1,-1)$ resonance $(\dot{\Phi} = -\dot{\omega})$. The accuracy of the first pair, $(q,k) = (0,2)$, is much better than has previously been obtained for order 29 from resonance analysis, or any other method (as Table 4 shows).

10 CONCLUSIONS

The orbit of 1968-40B has been determined at 74 epochs from some 6300 observations, between 1983 January and 1985 July while the satellite was passing through the condition of 29:2 resonance. The average accuracy of the inclination and eccentricity for all 74 epochs was equivalent to 110 m and 80 m in distance respectively.

The variations in inclination and eccentricity have been analysed, and six 29th-order lumped harmonics have been evaluated: the recommended values are those given in equation (9). The first two have standard deviations equivalent to accuracies of 0.3 and 0.6 cm in geoid height, considerably better than has

previously been obtained for order 29 from resonance analysis or any other method. The standard deviations of the other four values correspond to accuracies between 1.2 and 3.5 cm in geoid height. The results provide an independent test of the 29th-order harmonics from recent comprehensive gravity models: comparisons with five such models (Table 4) show good agreement and suggest that the estimated errors of the models are quite realistic, for order 29.

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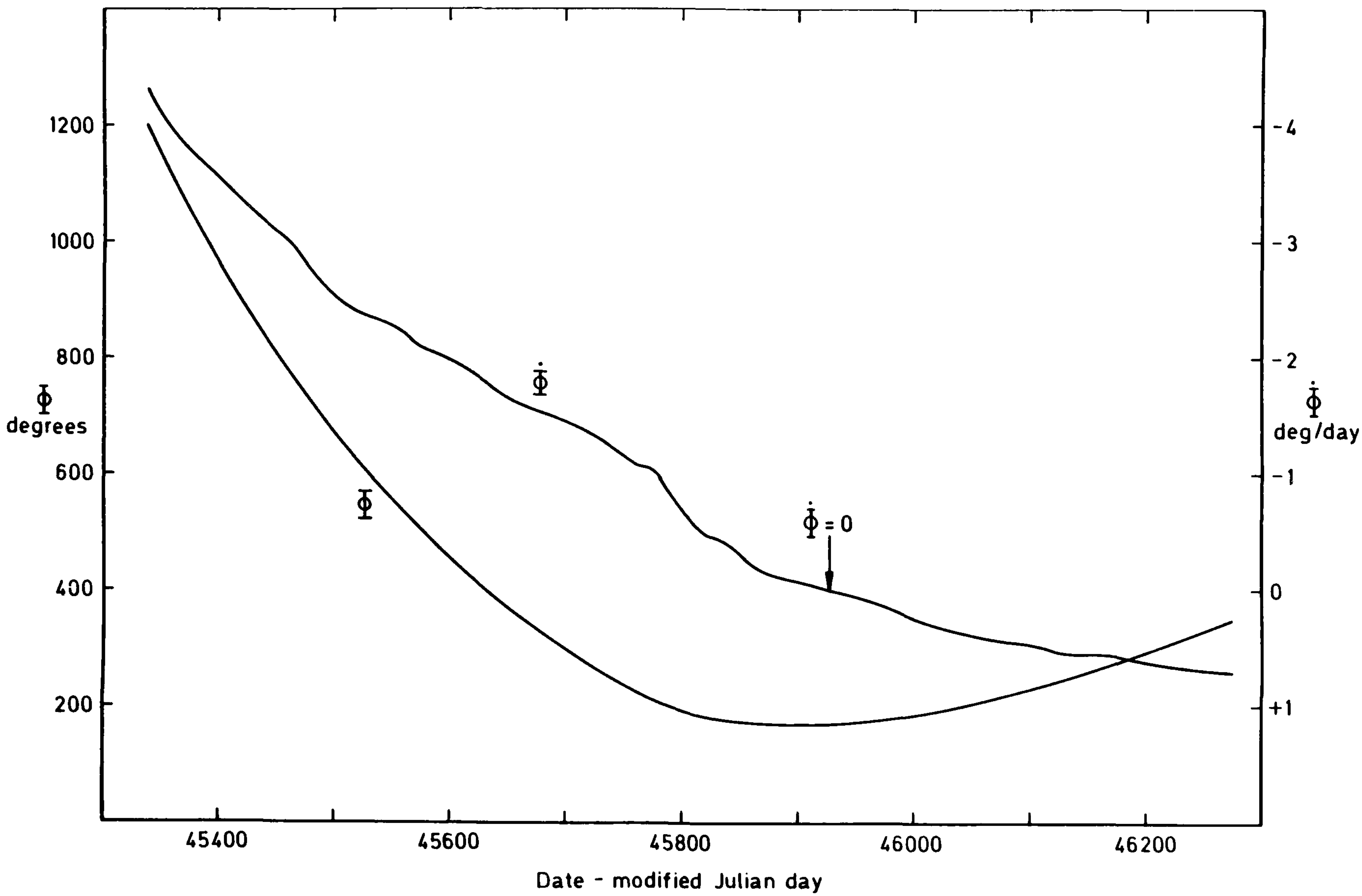

Fig 1 Variation of Φ and $\dot{\Phi}$ near 29:2 resonance

Fig 2

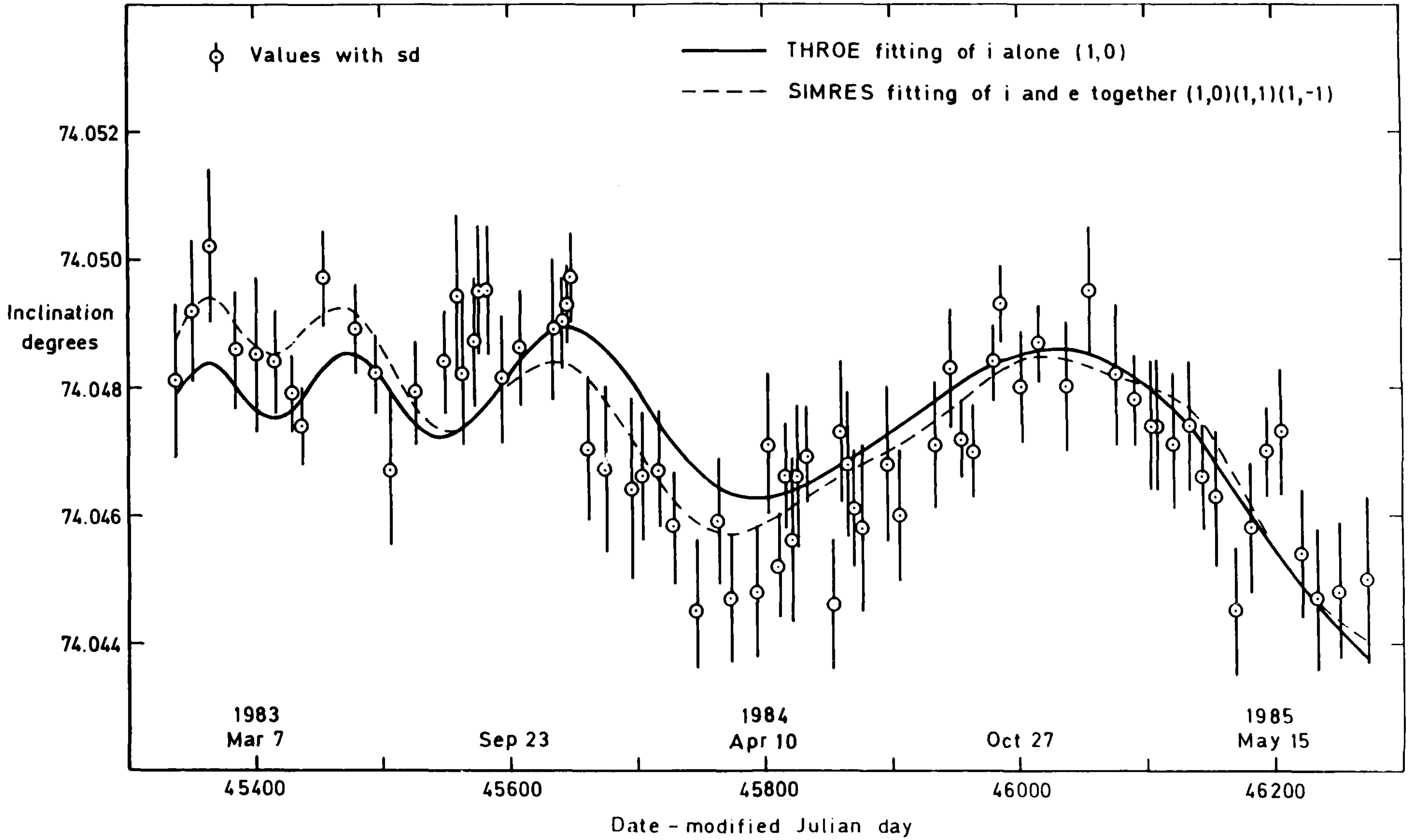

Fig 2 Values of inclination near 29:2 resonance, with fitted theoretical curves

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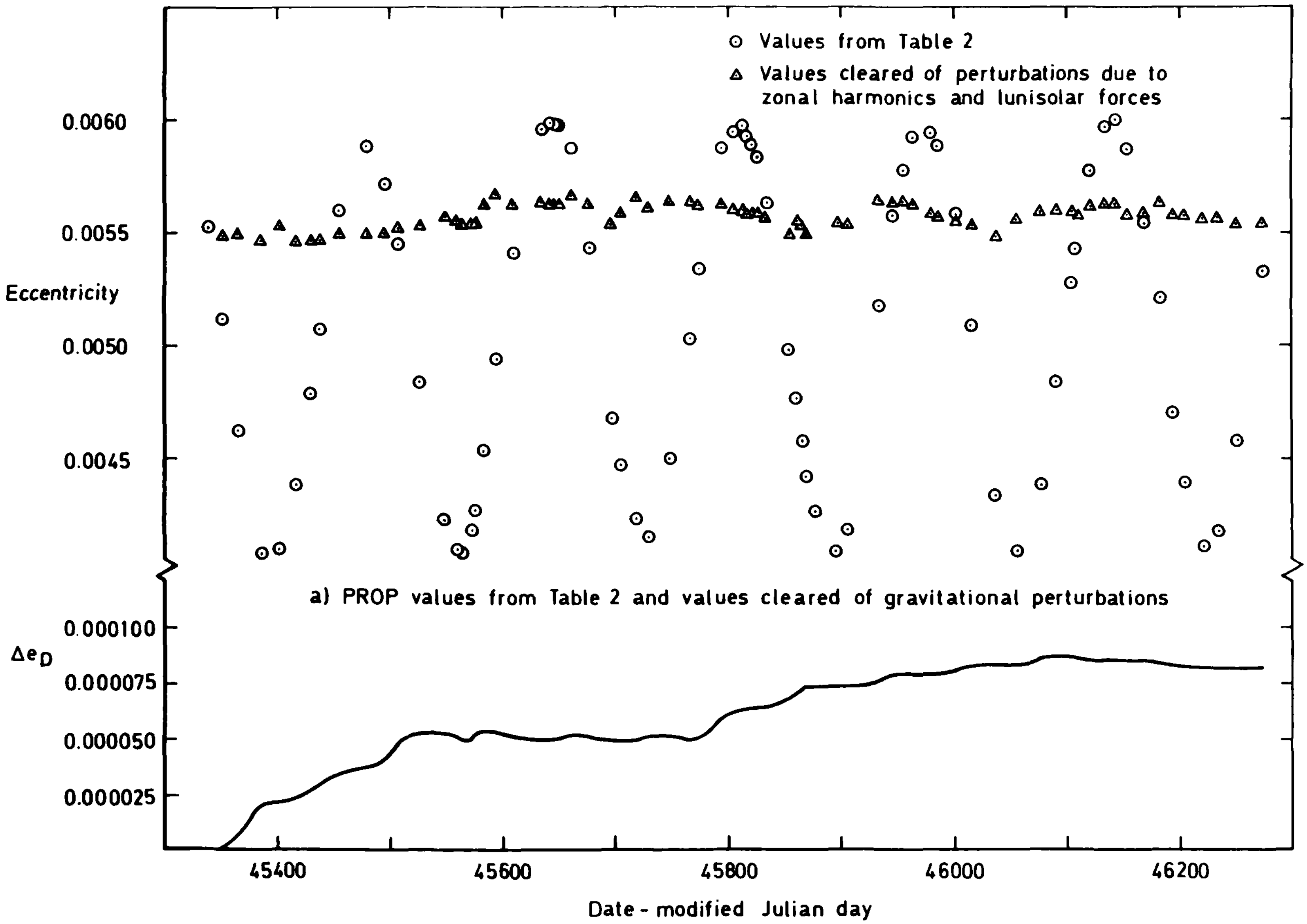

b) Correction to eccentricity due to drag in an atmosphere with day-to-night variation

Fig 3 Values of eccentricity and perturbations

Fig 4

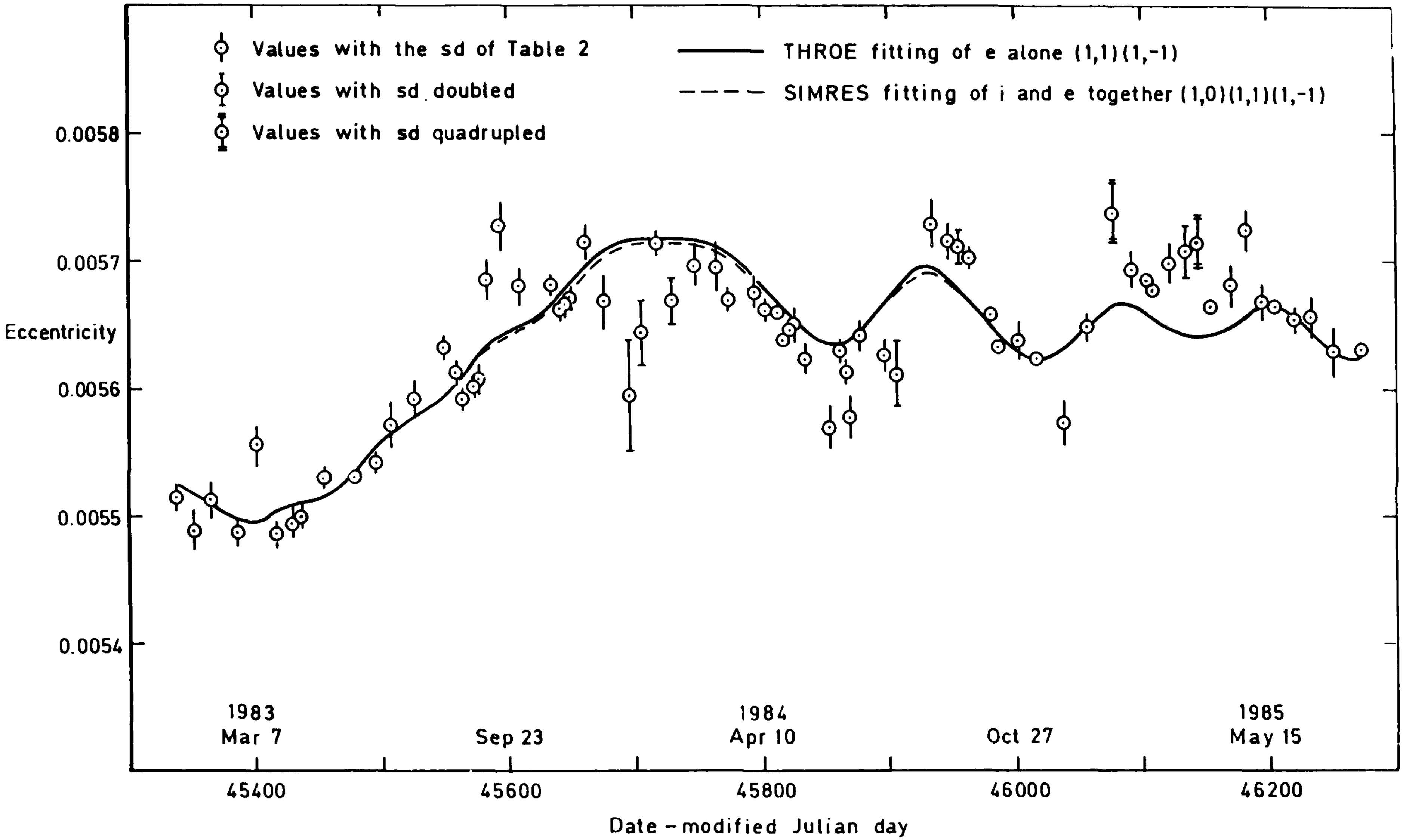

Fig 4 Values of eccentricity near 29:2 resonance, with fitted theoretical curves

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